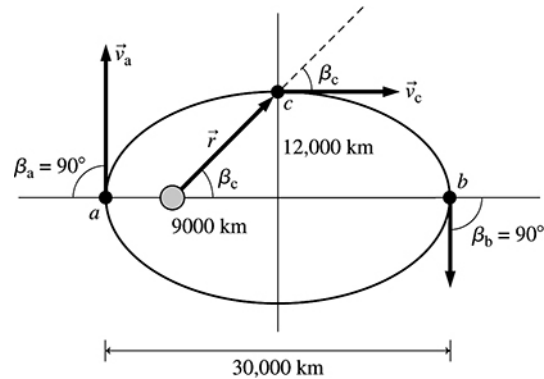


12.81. Model: The angular momentum of the satellite in the elliptical orbit is a constant.
Visualize:



Solve: (a) Because the gravitational force is always along the same direction as the direction of the moment arm vector, the torque $\vec{\tau} = \vec{r} \times \vec{F}_g$ is zero at all points on the orbit.

(b) The angular momentum of the satellite at any point on the elliptical trajectory is conserved. The velocity is perpendicular to \vec{r} at points a and b, so $\beta = 90^\circ$ and $L = mvr$. Thus

$$L_b = L_a \Rightarrow mv_b r_b = mv_a r_a \Rightarrow v_b = \left(\frac{r_a}{r_b} \right) v_a$$

$$r_a = \frac{30,000 \text{ km}}{2} - 9,000 \text{ km} = 6,000 \text{ km} \quad \text{and} \quad r_b = \frac{30,000 \text{ km}}{2} + 9,000 \text{ km} = 24,000 \text{ km}$$

$$\Rightarrow v_b = \left(\frac{6,000 \text{ km}}{24,000 \text{ km}} \right) (8,000 \text{ m/s}) = 2,000 \text{ m/s}$$

(c) Using the conservation of angular momentum $L_c = L_a$, we get

$$mv_c r_c \sin \beta_c = mv_a r_a \Rightarrow v_c = \left(\frac{r_a}{r_c} \right) v_a / \sin \beta_c \quad r_c = \sqrt{(9,000 \text{ km})^2 + (12,000 \text{ km})^2} = 1.5 \times 10^7 \text{ m}$$

From the figure, we see that $\sin \beta_c = 12,000/15,000 = 0.80$. Thus

$$v_c = \left(\frac{6,000 \text{ km}}{15,000 \text{ km}} \right) \frac{(8,000 \text{ m/s})}{0.80} = 4,000 \text{ m/s}$$